

# ANISOTROPIC QUANTUM COSMOLOGICAL MODELS: A DISCREPANCY BETWEEN MANY-WORLDS AND dBB INTERPRETATIONS

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## Abstract

In the isotropic quantum cosmological perfect fluid model, the initial singularity can be avoided, while the classical behaviour is recovered asymptotically. We verify if initial anisotropies can also be suppressed in a quantum version of a classical anisotropic model where gravity is coupled to a perfect fluid. Employing a Bianchi I cosmological model, we obtain a "Schrödinger-like" equation where the matter variables play the role of time. This equation has a hyperbolic signature. It can be explicitly solved and a wave packet is constructed. The expectation value of the scale factor, evaluated in the spirit of the many-worlds interpretation, reveals an isotropic Universe. On the other hand, the bohmian trajectories indicate the existence of anisotropies. This is an example where the Bohm-de Broglie and the many-worlds interpretations are not equivalent. It is argued that this inequivalence is due to the hyperbolic structure of the "Schrödinger-like" equation.

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## 1 Introduction

One of the main hopes regarding quantum cosmology is the possibility to obtain the initial conditions that will determine the ulterior evolution of the Universe when its classical regime is reached [1, 2]. For example, it is expected that the isotropy and homogeneity can be achieved due to the existence of a quantum phase, prior to the actual classical phase. Moreover, one expects that the initial singularity problem, which is one of the main drawback of the standard cosmological scenario, can be circumvented by the presence of quantum effects. In fact, it has already been shown explicitly that the classical singularity disappears in quantum cosmological scenario where gravity is coupled to a perfect fluid [3, 4, 5, 6]. In the present work, we address ourselves to another important question concerning the initial conditions of the Universe: can quantum effects suppress initial anisotropies, leading to an isotropic Universe as soon as the classical regime is reached?

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This is not the first time this question is addressed, of course, (see, for example, [7, 8, 9]). However, our approach distinguishes from the previous ones by the description employed for the perfect fluid, which allows in principle the identification of a time coordinate. It permits, as consequence, to compare in detail different interpretations schemes, like the many-worlds [10] and the Bohm-de Broglie (dBB) [11, 12] ones in order to obtain specific previsions for the evolution of the Universe and of the initial anisotropies. We will verify that there is a serious discrepancy between the results obtained through one scheme and the other, one showing an always isotropic Universe, while the other one indicates the existence of initial anisotropies which disappear asymptotically, as it happens in the corresponding classical model. This discrepancy reveals that the quantization of gravity coupled to perfect fluid leads to a very particular quantum system where some aspects of more ordinary systems, like the equivalence of the many-worlds and ontological approaches to quantum mechanics, are lost.

It would be perhaps worthfull to make some general considerations before to describe more in detail the problem to be treated here. It must be stressed, for example, that the meaning of the Wheeler-DeWitt equation and of the corresponding wave function of the Universe, which are the basic entities in quantum cosmology, have no consensus until now. Besides the technical problems connected with the Wheeler-DeWitt equation, a functional equation formulated in the superspace, the space of all possible three-dimensional metrics where no explicit time variable appears[13](what constitutes, of course, a serious problem), the interpretation of the results which can be obtained in specific situations is yet an open subject. Given a specific configuration (gravity plus perfect fluid, for example, as it is the case here), it is possible in principle to find its corresponding wave function, for example by freezing an infinity of degrees of freedom. This minisuperspace approach sacrifices of course some important aspects of the Wheeler-DeWitt equation, like its functional character. But, even if the Wheeler-DeWitt equation may be solved under some restrictive assumptions like the use of a minisuperspace, it remains the question of how to extract unambiguously predictions from the wave function of the Universe.

In spite of all the controversies, quantum cosmology has experienced some progress in these last years. As an example, it has been shown by Brown and Kuchar [14] that the coupling of gravity to a dust fluid can lead to the recovering of the notion of time at quantum level. The Wheeler-DeWitt equation can be recast in a form similar to that of the Schrödinger equation, and a Hilbert space structure can be constructed. More generally, when a perfect fluid is coupled to gravity, and degrees of freedom are attributed to the fluid by expressing it through some suitable set of variables, as it happens in the Schutz's formalism for the description of perfect fluid [15, 16], a Schrödinger-type equation can be obtained, with the fluid variables playing the role of time. The expectation value of some dynamical quantities can be evaluated, in the spirit of the so-called many-worlds interpretation, and a quantum scenario can be sketched. Alternatively, the bohmian trajectories can be computed.

Such approach has obviously some limitations. In the quantum regime, we could expect that fundamental fields should be considered, instead of a perfect fluid. Moreover, exact solutions can be obtained only in the minisuperspace, where just some degrees of freedom are kept. But, such quantum perfect fluid models can be, in spite of their

limitations, a very interesting laboratory, in the sense that the role played by quantum effects in the early Universe can be estimated. In a subject technically difficult and conceptually so controversial, the possibility of identifying naturally a time coordinate and of obtaining an explicit behaviour for some dynamical quantities, like the scale factor, may be seen as quite important achievements, in spite of the limitations of the model for the reasons quoted before.

Many aspects of the isotropic quantum cosmological perfect fluid models in the minisuperspace have been investigated in [3, 4, 5, 6, 17, 18, 19]. In particular, it has been shown that, while the corresponding classical models exhibit an initial singularity, the quantum scenario is non singular. The results are rigorously the same for the evaluation of the expectation values and of the bohmian trajectories. This seems to be a quite relevant result since it is expected that quantum cosmology can give answer to the question of what are the initial conditions for the Universe. Here, as it has already been said, we want to answer another question concerning the initial conditions of the Universe: to what extent anisotropies that may appear in a classical model can be washed out by quantum effects.

In some previous work regarding this question, anisotropic models coupled to a scalar field were studied. In [8, 9] a Bianchi I model was considered. In [8], the emphasis was on the tunneling from a classical forbidden to a classical allowed region in the configuration space. In [9] the bohmian trajectories were computed, revealing situations where isotropisation occurs. In [7] all other Bianchi models were taken into account in view of the evaluation of tunneling process. But, in all these cases there was no natural time coordinate in the Wheeler-DeWitt equation in the minisuperspace, and this fact limits drastically the extraction of unambiguous previsions from the quantum model.

As in the isotropic case, we will consider here an anisotropic model coupled to a perfect fluid, which is described with the aid of the Schutz's variables, what lead to a natural time coordinate connected with the fluid variables. In order to be specific, we will consider a Bianchi I cosmological model. Through a convenient choice of metric variables, the hamiltonian can be diagonalized. The Wheeler-DeWitt equation in the minisuperspace can be solved. A wave packet is constructed, satisfying the necessary boundary conditions. With this wave packet, and since the notion of time has been recovered by identifying it with the matter variables, we can try to extract some specific scenario from this model. As it has already been stressed, we can do it essentially in two ways: by evaluating the expectation value of the scale factor, in the spirit of the many-worlds interpretation of quantum mechanics, or by computing the bohmian trajectories, in the sense of the ontological interpretation of quantum mechanics. However, these two ways of obtaining a prediction (which are in principle connected to different interpretation's schemes) lead to different results: in the first one, we obtain that the Universe must be always isotropic; in the second one, anisotropies are present.

Perhaps this unexpected result can be understood by noting that the "Schrödinger-like" equation for the anisotropic perfect fluid cosmological model has a hyperbolic signature in its reduced hamiltonian. It constitutes, in this sense, a very unusual quantum system for which the equivalence between the many-worlds and ontological interpretations of quantum mechanics is lost, as it will be discussed more in detail later.

In the next section, we construct the Wheeler-DeWitt equation for the anisotropic perfect fluid model. In section 3, we determine the wave function by using the separation of variables method. In section 4, the expectation value of the scale factor and the bohmian trajectories are obtained. The discrepancy between them are settled out. In section 5 we discuss the results and present our conclusions.

## 2 Wheeler-DeWitt equation for an anisotropic perfect fluid model

Our starting point is the action of gravity coupled to a perfect fluid in the Schutz's formalism:

$$S = \int_M d^4x \sqrt{-g} R + 2 \int_{\partial M} d^3x \sqrt{h} h_{ab} K^{ab} + \int_M d^4x \sqrt{-g} p \quad (1)$$

where  $K^{ab}$  is the extrinsic curvature, and  $h_{ab}$  is the induced metric over the three-dimensional spatial hypersurface, which is the boundary  $\partial M$  of the four dimensional manifold  $M$ ; the factor  $16\pi G$  is made equal to one. The first two terms were first obtained in [2]; the last term of (1) represents the matter contribution to the total action in the Schutz's formalism for perfect fluids,  $p$  being the pressure, which is linked to the energy density by the equation of state  $p = \alpha\rho$ . In the Schutz's formalism [15, 16], the four-velocity is expressed in terms of five potentials  $\epsilon$ ,  $\zeta$ ,  $\beta$ ,  $\theta$  and  $S$ :

$$U_\nu = \frac{1}{\mu}(\epsilon_{,\nu} + \zeta\beta_{,\nu} + \theta S_{,\nu}) \quad (2)$$

where  $\mu$  is the specific enthalpy. The variable  $S$  is the specific entropy, while the potentials  $\zeta$  and  $\beta$  are connected with rotation and are absent for FRW's type models. The variables  $\epsilon$  and  $\theta$  have no clear physical meaning. The four velocity is subject to the condition

$$U^\nu U_\nu = 1 \quad . \quad (3)$$

The metric describing a Bianchi I anisotropic model is given by

$$ds^2 = N^2 dt^2 - \left( X(t)^2 dx^2 + Y(t)^2 dy^2 + Z(t)^2 dz^2 \right) \quad . \quad (4)$$

In this expression,  $N(t)$  is the lapse function. Using the constraints for the fluid, and after some thermodynamical considerations, the final reduced action, where surface terms were discarded, takes the form

$$S = \int dt \left[ -\frac{2}{N} \left( \dot{X}\dot{Y}Z + \dot{X}\dot{Z}Y + \dot{Y}\dot{Z}X \right) + N^{-1/\alpha} (XYZ) \frac{\alpha}{(\alpha+1)^{1/\alpha+1}} (\dot{\epsilon} + \theta\dot{S})^{1/\alpha+1} \exp\left(-\frac{S}{\alpha}\right) \right] \quad . \quad (5)$$

At this point, is more suitable to redefine the metric coefficients as

$$X(t) = e^{\beta_0 + \beta_+ + \sqrt{3}\beta_-} \quad , \quad Y(t) = e^{\beta_0 + \beta_+ - \sqrt{3}\beta_-} \quad , \quad Z(t) = e^{\beta_0 - 2\beta_+} \quad . \quad (6)$$

Using these new variables, the action may be simplified further, leading to the gravitational lagrangian density

$$L_G = -6 \frac{e^{3\beta_0}}{N} \{ \dot{\beta}_0^2 - \dot{\beta}_+^2 - \dot{\beta}_-^2 \} \quad . \quad (7)$$

From this expression, we can evaluate the conjugate momenta:

$$p_0 = -12 \frac{e^{3\beta_0}}{N} \dot{\beta}_0 \quad , \quad p_+ = 12 \frac{e^{3\beta_0}}{N} \dot{\beta}_+ \quad , \quad p_- = 12 \frac{e^{3\beta_0}}{N} \dot{\beta}_- \quad . \quad (8)$$

The matter sector may be recast in a more suitable form through the canonical transformations

$$T = p_S e^{-S} p_\epsilon^{-(\alpha+1)} \quad , \quad \Pi_T = p_\epsilon^{\alpha+1} e^S \quad , \quad \bar{\epsilon} = \epsilon - (\alpha+1) \frac{p_S}{p_\epsilon} \quad , \quad \bar{p}_\epsilon = p_\epsilon \quad . \quad (9)$$

The final expression for the total hamiltonian is

$$H = N e^{-3\beta_0} \left\{ -\frac{1}{24} (p_0^2 - p_+^2 - p_-^2) + e^{3(1-\alpha)\beta_0} p_T \right\} \quad . \quad (10)$$

The lapse function  $N$  plays the role of a lagrangian multiplier in (10). It leads to the constraint

$$H = 0 \quad . \quad (11)$$

The quantization procedure consists in considering the hamiltonian as an operator which is applied on a wave function

$$\hat{H}\psi = 0 \quad (12)$$

taking at the same time the momenta as operator, in the present case in the coordinate representation (we use natural units where  $\hbar = 1$ ):

$$\hat{p}_i = -i \frac{\partial}{\partial \beta_i} \quad . \quad (13)$$

Since the momentum associated to the matter degrees of freedom appears linearly in the hamiltonian, we can identify it with a time coordinate

$$\hat{p}_T = -i \frac{\partial}{\partial T} \quad . \quad (14)$$

Due to the canonical transformations employed before, this new time is related to the cosmic time  $t$  by  $dt = e^{3\alpha\beta_0} dT$ . In this way, we end up with the Wheeler-DeWitt equation, in the minisuperspace, for an anisotropic Universe filled with a perfect fluid:

$$\left( \frac{\partial^2}{\partial \beta_0^2} - \frac{\partial^2}{\partial \beta_+^2} - \frac{\partial^2}{\partial \beta_-^2} \right) \psi = 24i e^{3(1-\alpha)\beta_0} \frac{\partial \psi}{\partial T} \quad . \quad (15)$$

### 3 Construction of the wave packet

Now, our goal is to solve (15) and to construct the corresponding wave packet. To do so, we use the separation of variable method. First, we write the wave function as

$$\psi(\beta_0, \beta_+, \beta_-, T) = \phi(\beta_0, \beta_+, \beta_-) e^{-iET} \quad , \quad (16)$$

leading to the equation

$$\left( \frac{\partial^2}{\partial \beta_0^2} - \frac{\partial^2}{\partial \beta_+^2} - \frac{\partial^2}{\partial \beta_-^2} \right) \phi = 24E e^{3(1-\alpha)\beta_0} \phi \quad . \quad (17)$$

The function  $\phi$  is then written as

$$\phi(\beta_0, \beta_+, \beta_-) = \Upsilon_0(\beta_0) \Upsilon_+(\beta_+) \Upsilon_-(\beta_-) \quad , \quad (18)$$

leading to the equation

$$\frac{\partial_0^2 \Upsilon_0}{\Upsilon_0} + 24E e^{3(1-\alpha)\beta_0} - \frac{\partial_+^2 \Upsilon_+}{\Upsilon_+} - \frac{\partial_-^2 \Upsilon_-}{\Upsilon_-} = 0 \quad (19)$$

where we have simplified in a obvious way the notation for the partial derivatives. The natural ansatz for the functions  $\Upsilon_{\pm}$  is

$$\Upsilon_{\pm} = C_{\pm} e^{ik_{\pm}\beta_{\pm}} \quad , \quad (20)$$

where  $C_{\pm}$  are constants and  $k_{\pm}$  are the separation parameters. This separation parameters must be real otherwise the wave function is not normalizable.

The equation determining the behaviour of  $\Upsilon_0$  takes then the form,

$$\Upsilon_0'' + \left( 24E e^{3(1-\alpha)\beta_0} + (k_+^2 + k_-^2) \right) \Upsilon_0 = 0 \quad , \quad (21)$$

the primes meaning derivatives with respect to  $\beta_0$ . It is easily to see that the parameter  $E$  must be positive. The previous equation can be solved through the redefinitions

$$a = e^{\beta_0} \quad , \quad y = a^r \quad , \quad r = \frac{3}{2}(1-\alpha) \quad . \quad (22)$$

after what (21) takes the form of a Bessel's equation:

$$\ddot{\Upsilon}_0 + \frac{\dot{\Upsilon}_0}{y} + \left( \frac{24E}{r^2} + \frac{k^2}{r^2} \frac{1}{y^2} \right) \Upsilon_0 = 0 \quad (23)$$

where  $k^2 = k_+^2 + k_-^2$  and the dots are derivatives with respect to  $y$ . The solution is

$$\Upsilon_0 = C_1 J_{\nu} \left( \frac{\sqrt{24E}}{r} a^r \right) + C_2 J_{-\nu} \left( \frac{\sqrt{24E}}{r} a^r \right) \quad , \quad (24)$$

with  $\nu = ik/r$ ,  $C_{1,2}$  being integration constants.

The final expression for the wave function is then

$$\Psi = e^{i(k_+\beta_++k_-\beta_-)} \left[ \bar{C}_1 J_\nu \left( \frac{\sqrt{24E}}{r} a^r \right) + \bar{C}_2 J_{-\nu} \left( \frac{\sqrt{24E}}{r} a^r \right) \right] e^{-iET} \quad (25)$$

where  $\bar{C}_{1,2}$  are combinations of the preceding integration constants. We want now to construct a superposition of these solutions, generating a regular wave packet. In principle, this can be achieved by considering the integration constants as gaussian functions of the integration parameters  $k_\pm$  and  $E$ . The general case constitutes a hard problem from the technical point of view. However, since the variables  $\beta_+$  and  $\beta_-$  appear in a symmetric form in (15), we may consider, for simplicity, the final wave function as independent of one of them, which amounts to fix one the corresponding separation parameter  $k_+$  or  $k_-$  equal to zero. From here on we will consider  $k_- = 0$ . Notice that the final results would be the same if we had imposed  $k_+ = 0$  and  $k_- \neq 0$ . Hence, even if the anisotropic models are not analyzed in all their generality, a large class of them is covered in what follows.

Fixing  $k_- = 0$ , the wave packet is given by

$$\Psi = \int e^{ik_+\beta_+} \left\{ \bar{C}_1 J_\nu \left( \frac{\sqrt{24E}}{r} a^r \right) + \bar{C}_2 J_{-\nu} \left( \frac{\sqrt{24E}}{r} a^r \right) \right\} e^{-iET} dk_+ dE \quad . \quad (26)$$

In principle, in the expression for  $\nu$  it appears the modulus of  $k_+$  while in the first exponential in (26) we have  $-\infty < k_+ < +\infty$ . We will consider a superposition of both Bessel's functions in such a way that the expression for the wave packet may be written as

$$\Psi = \int_{-\infty}^{+\infty} \int_0^\infty A(k_+, q) e^{ik_+\beta_+} J_\nu \left( q a^r \right) e^{-iq^2 T} dk_+ dq \quad , \quad (27)$$

with  $q = \frac{\sqrt{24E}}{r}$  and

$$A(k_+, q) = e^{-\gamma k_+^2} q^{\nu+1} e^{-\lambda q^2} \quad . \quad (28)$$

In this case, the integrals can be explicitly calculated, leading to the wave packet

$$\Psi = \frac{\Psi_0}{B} \exp \left[ -\frac{a^{2r}}{4B} - \frac{(\beta_+ + C(a, \beta_+))^2}{4\lambda} \right] \quad (29)$$

where  $\Psi_0$  is a constant and

$$B = \lambda + isT \quad , \quad C(a, \beta_+) = \ln a - \frac{2}{3(1-\alpha)} \ln B \quad , \quad s = -\frac{3(1-\alpha)^2}{32} \quad . \quad (30)$$

Notice that the wave packet given by (29) is square integrable, and it vanishes in the extremes of the interval of validity of the variables  $a = e^{\beta_0}$  and  $\beta_+$ , being consequently regular as it is physically required. Remark that the wave packet (29) is indeed a solution of the equation (15), as it can be explicitly verified.

## 4 The scenario for the Universe

Having the expression for the wave function of the Universe, it is time now to obtain a specific prediction for the behaviour of the dynamical functions in this model. To do so, there is two options: to evaluate the expectation values of the functions describing the evolution of the scale factors, in the spirit of the many-worlds interpretation of quantum mechanics; to evaluate the bohmian trajectories for those functions, in the realm of the ontological interpretation of quantum mechanics. In all known cases, these different procedures lead to the same results (to an explicit example treating the isotropic version of the present model, see [18]). The former procedure is possible in our case since we have a time coordinate  $T$ . We will evaluate the behaviour of the functions  $\beta_0$  and  $\beta_+$  using these two procedures.

Before to do this, let us just recall the classical solutions for the Bianchi I cosmological model with a barotropic perfect fluid described by  $p = \alpha\rho$ . For the time parametrization  $dt = a^{3\alpha} dT$ ,  $t$  being the cosmic time, the functions  $X$ ,  $Y$ , and  $Z$  admit the solution

$$X(T) = e^{\beta_0 + \beta_+ + \sqrt{3}\beta_-} = X_0 \left( T + c \right)^{\frac{1+2s_1}{3(1-\alpha)}} \left( T - c \right)^{\frac{1-2s_1}{3(1-\alpha)}} , \quad (31)$$

$$Y(T) = e^{\beta_0 + \beta_+ - \sqrt{3}\beta_-} = Y_0 \left( T + c \right)^{\frac{1+2s_2}{3(1-\alpha)}} \left( T - c \right)^{\frac{1-2s_2}{3(1-\alpha)}} , \quad (32)$$

$$Z(T) = e^{\beta_0 - 2\beta_+} = Z_0 \left( T + c \right)^{\frac{1+2s_3}{3(1-\alpha)}} \left( T - c \right)^{\frac{1-2s_3}{3(1-\alpha)}} , \quad (33)$$

where  $c$  is constant, and  $s_1$ ,  $s_2$  and  $s_3$  are parameters such that

$$s_1 + s_2 + s_3 = 0 \quad , \quad s_1^2 + s_2^2 + s_3^2 = 6 \quad . \quad (34)$$

Notice that there is an initial singularity, near which the Universe is very anisotropic, becoming isotropic asymptotically.

Let us return now to the computation of the quantum scenario through the use of the many-worlds and ontological interpretations of quantum mechanics.

### 4.1 Expectation values of the dynamical variables

Given the wave function  $\Psi$ , the expectation value of a variable  $\beta_i$  is obtained in the usual way:

$$\langle \beta_i \rangle = \frac{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{3(1-\alpha)\beta_0} \Psi^* \beta_i \Psi d\beta_0 d\beta_+}{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{3(1-\alpha)\beta_0} \Psi^* \Psi d\beta_0 d\beta_+} . \quad (35)$$

The unusual measure in the integrals is due to the requirement that the reduced hamiltonian in (15) must be symmetric [18, 19].

The first step is to calculate the denominator, which will be a common term for the computation of  $\langle \beta_0 \rangle$  and  $\langle \beta_+ \rangle$ . Using again the definition  $a = e^{\beta_0}$  and integrating in  $\beta_+$ , we obtain

$$\int_0^\infty \int_{-\infty}^\infty a^{2-3\alpha} \Psi^* \Psi da d\beta_+ = \sqrt{2\gamma\pi} F(T) \int_0^\infty a^{2-3\alpha} \exp\left(-\frac{\lambda a^{3(1-\alpha)}}{2B^*B}\right) da \quad , \quad (36)$$

where

$$F(T) = \frac{\exp(\frac{C^2}{2\gamma})}{B^*B} \quad , \quad (37)$$

and

$$\begin{aligned} C(a, \beta_+) &= C_R + iC_I \quad , \\ C_R &= \ln a - \frac{1}{3(1-\alpha)} \ln 4B^*B \quad , \quad C_I = \frac{-2}{3(1-\alpha)} \arctan\left(\frac{sT}{\lambda}\right) \quad . \end{aligned} \quad (38)$$

On the other hand, with  $\beta_i = \beta_0$  in (35) we find for the numerator:

$$\int_0^\infty a^{2-3\alpha} \Psi^* \Psi \ln a \, da \, d\beta_+ = \frac{F(T) \sqrt{2\gamma\pi}}{9(1-\alpha)^2} \left[ \frac{2B^*B}{\lambda} \right] \left\{ \ln\left(\frac{2B^*B}{\lambda}\right) + n \right\} \quad , \quad (39)$$

where we have noted

$$n = \int_0^\infty \exp(-u) \ln u \, du \sim -0.577 \quad , \quad u = \frac{\lambda}{2B^*B} a^{3(1-\alpha)} \quad . \quad (40)$$

Hence,

$$< \beta_0 > = \frac{1}{3(1-\alpha)} \left\{ \ln\left(\frac{2|B|^2}{\lambda}\right) + n \right\} \quad . \quad (41)$$

This result leads to

$$e^{<\beta_0>} = (XYZ)^{1/3} = a_0 \left[ 1 + \frac{s^2 T^2}{\lambda^2} \right]^{\frac{1}{3(1-\alpha)}} \quad , \quad (42)$$

where  $a_0$  is a constant. This is the same result as in the isotropic case [18]. Consequently, the space volume evolves as in the corresponding isotropic case.

So, the anisotropies must be represented by the function  $\beta_+$ , whose expectation value will be computed in what follows. We will evaluate now the numerator of (35) with  $\beta_i = \beta_+$ . Integrating in  $\beta_+$  and expressing  $\beta_0$  in terms of  $a$  as before, we find:

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{3(1-\alpha)} \Psi^* \beta_+ \Psi \, d\beta_0 \, d\beta_+ = -\sqrt{\pi} \left\{ I_1 - \frac{\ln(4B^*B)}{3(1-\alpha)} I_2 \right\} F(T) \quad , \quad (43)$$

$$I_1 = \int_0^\infty a^{2-3\alpha} \exp\left\{ -\lambda \frac{a^{3(1-\alpha)}}{2B^*B} \right\} \ln a \, da \quad , \quad I_2 = \int_0^\infty a^{2-3\alpha} \exp\left\{ -\lambda \frac{a^{3(1-\alpha)}}{2B^*B} \right\} da \quad (44)$$

The integrals  $I_1$  and  $I_2$  take the form,

$$I_1 = \frac{1}{9(1-\alpha)^2} \left[ \frac{2B^*B}{\lambda} \right] \left[ n + \ln\left(\frac{2B^*B}{\lambda}\right) \right] \quad , \quad I_2 = \frac{1}{3(1-\alpha)} \frac{2B^*B}{\lambda} \quad . \quad (45)$$

Using the previous result for the denominator of (35) we find finally

$$< \beta_+ > = \frac{1}{3(1-\alpha)} \frac{1}{\sqrt{2\gamma}} \left\{ \ln(2\lambda) - n \right\} \quad . \quad (46)$$

Surprisingly, the expectation value of  $\beta_+$  does not depend on time. Consequently, the predicted result for the evolution of the Universe in this case is exactly the same as in the isotropic case: there is no anisotropy during all the evolution of the Universe.

## 4.2 Computation of the bohmian trajectories

The result found in the last section is quite unexpected. There is no trace of the anisotropies existing in the classical model in the corresponding quantum analysis. We note that in (15), the variables  $\beta_+$  and  $\beta_-$  appear symmetrically, what it is not the case in the classical model. But, in order to verify better the meaning of this result, we will evaluate the bohmian trajectories which determine the behaviour of a quantum system in the ontological interpretation of quantum mechanics. In principle, the results furnished by the bohmian trajectories must be equivalent to those obtained through the computation of the expectation values.

In the ontological interpretation of quantum mechanics, the wave function is written as

$$\Psi = R \exp(iS) \quad , \quad (47)$$

where  $R$  is connected with the amplitude of the wave function, and  $S$  to its phase. When (47) is inserted in the Schrödinger's equation, the real and imaginary parts of the resulting expression leads to the conservation of probability and to a Hamilton-Jacobi's equation supplemented by a term which is identified as the quantum potential, which leads to the quantum effects distinguishing the quantum trajectories from the classical ones.

In this formulation of quantum mechanics, the trajectories (which are real trajectories) corresponding to a dynamical variable  $q$  with a conjugate momentum  $p_q$ , are given by

$$\dot{p}_q = \frac{\partial S}{\partial q} \quad . \quad (48)$$

Hence, the ontological formulation of quantum mechanics leads to a natural identification of a time coordinate, what is very important for quantum cosmology where in general there is no explicit time coordinate.

Let us consider the wave function (29). Putting in the form (47), the phase reads,

$$S(\beta_0, \beta_+, T) = -\arctan\left(\frac{sT}{\lambda}\right) + \frac{sT a^{3(1-\alpha)}}{4B^*B} - \frac{C_I}{2\gamma}(\beta_+ + C_R) \quad , \quad (49)$$

where all quantities are defined as before. The conjugate momenta associated to the dynamical variables  $\beta_0$  and  $\beta_+$  read

$$p_0 = -12a^{2-3\alpha}\dot{a} \quad , \quad p_+ = 12a^{3(1-\alpha)}\dot{\beta}_+ \quad , \quad (50)$$

where we have explicitly used the time parametrization such that the lapse function is given by  $N = a^{3\alpha}$ . The bohmian trajectories are then given by the expressions

$$-12a^{2-3\alpha}\dot{a} = 3(1-\alpha)\frac{sT}{4B^*B}a^{3(1-\alpha)} - \frac{C_I}{2\gamma} \quad , \quad (51)$$

$$12a^{3(1-\alpha)}\dot{\beta}_+ = -\frac{C_I}{2\gamma} \quad , \quad (52)$$

dots representing derivatives with respect to  $T$ . Combining (51,52), we find

$$-12a^{2-3\alpha}\dot{a} = 3(1-\alpha)\frac{sT}{4B^*B}a^{3(1-\alpha)} + 12a^{3(1-\alpha)}\dot{\beta}_+ \quad . \quad (53)$$

This last equation lead after integration to the expression

$$ae^{\beta_+} = D \left[ \lambda^2 + s^2 T^2 \right]^{\frac{1}{3(1-\alpha)}} . \quad (54)$$

Reinserting the relation in the equations (51,52) we can obtain the following solutions to  $a$  and  $\beta_+$ :

$$a = \left( \frac{-1}{24s\lambda\gamma} \right)^{\frac{1}{3(1-\alpha)}} \left[ \lambda^2 + s^2 T^2 \right]^{\frac{1}{3(1-\alpha)}} \left[ \arctan^2 \left( \frac{sT}{\lambda} \right) + E \right]^{\frac{1}{3(1-\alpha)}} , \quad (55)$$

$$\beta_+ = -\frac{1}{3(1-\alpha)} \ln \left\{ \arctan^2 \left( \frac{sT}{\lambda} \right) + E \right\} + \ln \left\{ \left[ -24s\lambda\gamma \right]^{\frac{1}{3(1-\alpha)}} \right\} + \ln D , \quad (56)$$

where  $E$  and  $D$  are integration constants. Remember that  $s < 0$ .

In opposition to the expressions obtained for the expectation values of  $\beta_0$  (which is connected to  $a$ ) and  $\beta_+$  in the preceding subsection, the bohmian trajectories predict an anisotropic Universe. Until this point, this strange discrepancy is not so catastrophic: in order the bohmian trajectories coincide with the results for the expectation value for some quantity, the integration constants that appear in the former must be averaged over an initial distribution given by the modulo of the wave function at  $T = 0$ . At  $T = 0$ , we have

$$a(T=0) = \left( \frac{-\lambda E}{24s\gamma} \right)^{\frac{1}{3(1-\alpha)}} , \quad (57)$$

$$\beta_+(T=0) = \ln \left\{ \left[ \frac{-24s\lambda\gamma D^{3(1-\alpha)}}{E} \right]^{\frac{1}{3(1-\alpha)}} \right\} . \quad (58)$$

Hence,

$$\mathcal{R} = \Psi^* \Psi|_{T=0} = \frac{\Psi_0^2}{\lambda^2} e^{\frac{E}{48s\gamma} - \frac{1}{18(1-\alpha)^2\gamma} \ln^2 \left[ \frac{D^{3(1-\alpha)}}{4} \right]} . \quad (59)$$

For  $a$  and  $\beta_+$  the average on the initial conditions leads to the integral expressions

$$\bar{a}(T) = \int_0^\infty \int_0^\infty \mathcal{R} a(T) dE dD , \quad (60)$$

$$\bar{\beta}_+(T) = \int_0^\infty \int_0^\infty \mathcal{R} \beta_+(T) dE dD , \quad (61)$$

These expressions can be recast in the following form:

$$\bar{a}(T) = \beta_2 \left\{ -\frac{\lambda^2 + s^2 T^2}{24s\lambda\gamma} \right\}^{\frac{1}{3(1-\alpha)}} \int_0^\infty \exp \left[ \frac{x}{48s\lambda\gamma} \right] \left[ \arctan^2 \left( \frac{sT}{\lambda} \right) + x \right]^{\frac{1}{3(1-\alpha)}} dx , \quad (62)$$

$$\bar{\beta}_+ = \beta_1 - \frac{\beta_2}{3(1-\alpha)} \int_0^\infty \exp \left[ \frac{x}{48s\lambda\gamma} \right] \ln \left\{ \left[ \arctan^2 \left( \frac{sT}{\lambda} \right) + x \right] \right\} dx , \quad (63)$$

where  $\beta_{1,2}$  are numbers given by

$$\beta_1 = 48s\bar{\Psi}_0^2 \frac{\gamma}{\lambda} \int_0^\infty \exp \left[ -\frac{\ln^2 y}{2\gamma} \right] dy , \quad (64)$$

$$\beta_2 = \frac{\bar{\Psi}_0^2}{\lambda^2} \int_0^\infty \exp\left[-\frac{\ln^2 y}{2\gamma}\right] dy \quad , \quad (65)$$

$$\bar{\Psi}_0^2 = \Psi_0^2 \exp\left[\frac{1}{2\gamma} \frac{\ln 2}{9(1-\alpha)^2}\right] \quad . \quad (66)$$

In the expressions above, we have written  $x = E$  and  $y = D$ . The variables  $x$  and  $y$  were restricted to positive values in order to assure that the metric functions are real. Even if the integrals (62,63) seem to admit no simple closed expressions, it is evident that they are time dependent. The behaviour of  $\beta_+$  and  $a$  in function of time are displayed in figures 1 and 2 for  $\alpha = 0$  and  $\lambda = \gamma = 1$ , indicating the presence of anisotropies even after averaging on the initial conditions, and revealing a non singular scenario, as can be easily deduced from the above expressions. Notice that these anisotropies disappear in both asymptotes. Hence, the bohmian trajectories predict an anisotropic Universe even after averaging on the initial probability distribution, in disagreement with the result obtained through the computation of the expectation value of the functions  $\beta_0$  and  $\beta_+$ .

## 5 Conclusions

It is generally expected that quantum effects in the very early universe may furnish the set of initial conditions which will determine the subsequent evolution of the Universe when its classical phase is reached. By initial conditions we mean here the isotropy and homogeneity. Moreover, it is also expected that those quantum effects may lead to the avoidance of the initial singularity, one of the major problems of the standard cosmological model. In this work we have tried to analyse the possibility that quantum effects can suppress initial anisotropies. Specifically, we have studied a Bianchi I model with a perfect fluid, with an isotropic pressure, employing the Schutz's description for perfect fluids. This problem has for us two main interests: first, it adds more degrees of freedom with respect to the isotropic model, since now we have four independent variables instead of just two; it permits to verify if anisotropies in the early Universe disappear in the quantum model, as it happens with the initial singularity for the corresponding isotropic one. The employment of Schutz's formalism for the description of the perfect fluid present in the model allows us to identify quite naturally a time coordinate associated to the matter degrees of freedom, since the canonical momentum corresponding to the matter variables appears linearly in the hamiltonian. Hence, the Wheeler-DeWitt equation can be reduced to a Schrödinger-like equation.

We have solved the Wheeler-DeWitt equation in the minisuperspace. A wave packet was constructed for the special case that the wave function is independent of one of the variables, namely  $\beta_-$ . This restriction was made only because of technical reasons, since it permits to obtain a closed expression for the wave packet. This wave packet is regular in the sense that it is square integrable and it vanishes in the extreme of the intervals of  $\beta_0$  and  $\beta_+$ . Using this wave packet, we have determined the behaviour of the metric functions using first the many-worlds interpretation of quantum mechanics, which implies to compute the expectation value of those functions. We found that there is no trace of anisotropies at any moment: the expectation value of the function  $\beta_+$  is constant while

the expectation value of  $\beta_0$  has essentially the same expression as in the isotropic version of this problem. All the features of this model are the same as in the isotropic case.

Later, we have determined the behaviour of metric functions employing the ontological interpretation of quantum mechanics, determining the bohmian trajectories. Surprisingly, in this case the function  $\beta_+$  is no longer a constant, and an initial anisotropic Universe is predicted. Asymptotically, it becomes isotropic like in the classical case. This result is maintained even after the averaging on the initial conditions. As it is well known [11, 12] the bohmian trajectories should lead to the same results that are obtained computing the expectation values after averaging on the initial conditions. This equivalence does not occur for the anisotropic Bianchi I cosmological model.

To our knowledge, this is the first case where those interpretation schemes predict different results. In principle, since the ontological and the many-worlds interpretations of quantum mechanics are precisely "interpretations" procedures, they must furnish the same results which are "seen" from different perspective, as we have already stressed. In the present case, the results themselves are different. What are the reasons for this discrepancy? We guess that the reason for this unexpected feature lies in the fact that the "Schrödinger-like" equation obtained after quantizing the anisotropic perfect fluid model has a hyperbolic signature in its "spatial" sector. This leads to two different problems: first, the energy of a free-particle is not positive definite anymore; second, in some sense the functions  $\beta_+$  and  $\beta_-$  play also the role of time, in a way similar to what happens in the Klein-Gordon equation. For both reasons, the equivalence between the ontological and many-worlds approaches to the problem is broken. This rises the question of which approach to use, a very intriguing problem that it is not addressed in the present paper.

To verify the hypothesis that the discrepancy found in the results obtained using the many-worlds and ontological interpretations of quantum mechanic, we change the signature of the hamiltonian (10) by force, making the transformations  $p_+^2 \rightarrow -p_+^2$  and  $p_-^2 \rightarrow -p_-^2$ . In this way, the hamiltonian (10) gains an elliptical structure. In this case, discarding the momentum  $p_-$  by simplicity, what leads to two dimensional problem, and performing the canonical transformations

$$x = \sqrt{\frac{32}{3}} \frac{e^{\frac{3}{2}(1-\alpha)\beta_0}}{1-\alpha} \quad , \quad p_x = \frac{1}{24} p_0 e^{-\frac{3}{2}(1-\alpha)\beta_0} \quad , \quad (67)$$

$$y = \sqrt{24} e^{\frac{3}{2}(1-\alpha)\beta_0} \beta_+ \quad , \quad p_y = \frac{1}{24} p_+ e^{-\frac{3}{2}(1-\alpha)\beta_0} \quad , \quad (68)$$

the hamiltonian (10) takes the form

$$H = -p_x^2 - p_y^2 + p_T \quad , \quad (69)$$

which is the hamiltonian for a free-particle in two dimensions. In the free-particle problem, it is easy to verify that the many-worlds and ontological interpretations give the same results [11]. Hence, the discrepancy found above disappears if the the signature of the spatial part of the hamiltonian is made elliptical.

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## Figure captions

Figure 1: Behaviour of  $\beta_+$  for  $\alpha = 0$ ,  $\lambda = \gamma = 1$

Figure 2: Behaviour of  $a$  for  $\alpha = 0$ ,  $\lambda = \gamma = 1$